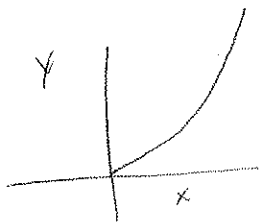


Geometry II

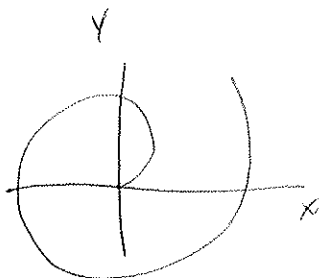
Solutions to Coursework #1

1. a)



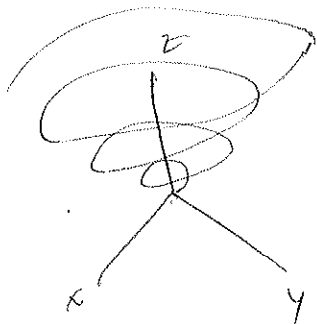
open curve

b)



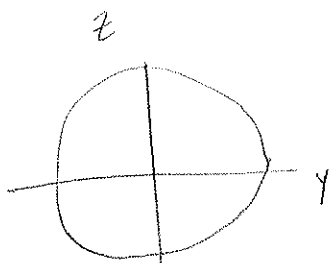
open curve

c)



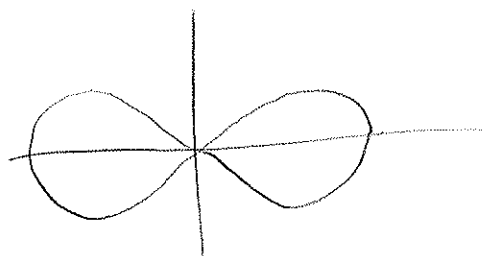
open curve

d)



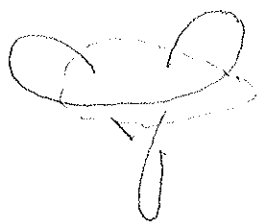
simple closed
curve of
period π

e)



closed, but
not simple
closed, curve

2.



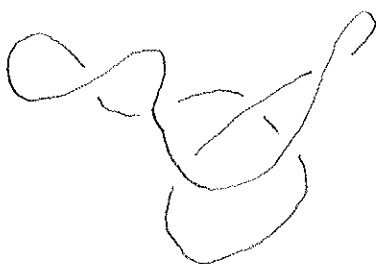
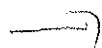
Type III



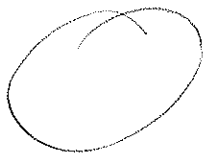
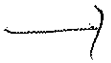
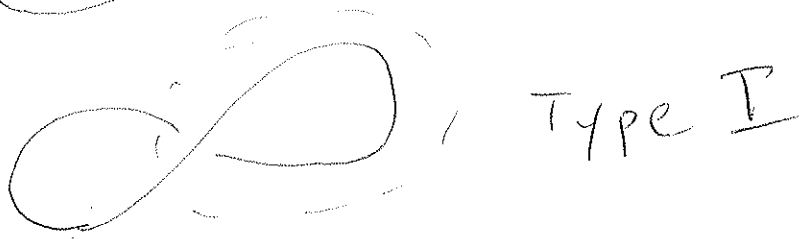
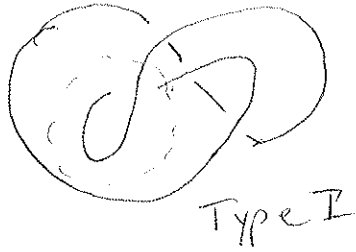
type I



Type II

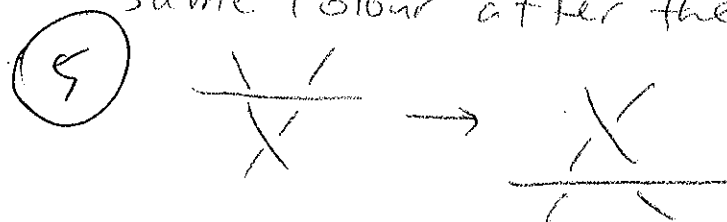


3.



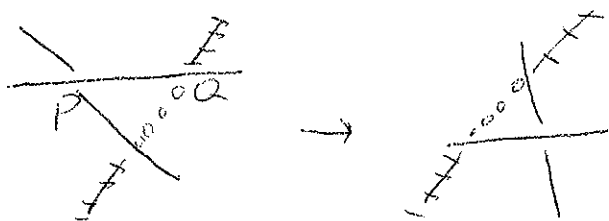
4. a) If a knot is tricoloured and a move III is made, 3 crossings are affected!

Case (1) If all three have same colour, then we use same colour after the move:



Case (2)

(10) If crossing P, say, had all one colour, then Q must have 3 colours (else we're back in case (1)) so



shows the knot is still tricolourable after the move. (Note: the ends of the arcs must not change colour under the move).

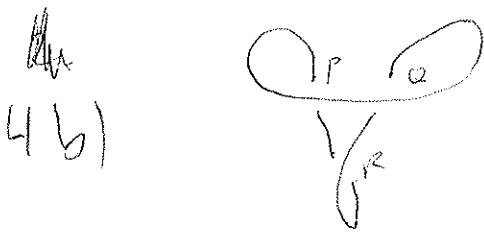
Case (3)

If P has 3 colours and so do others, then



shows the knot is still tricolourable after the move.

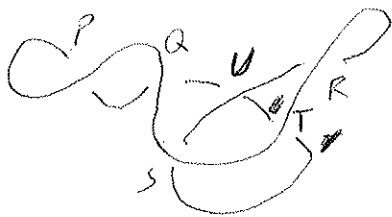
done in class



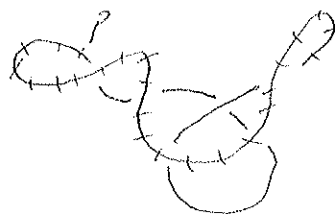
If P has only one colour (say blue), then Q has 2 arcs blue hence all three are blue, therefore only blue is used in the diagram and we cannot have a tri-colouring.

If P has 3 colours: this is impossible because one arc appears twice at P.

Therefore the left hand diagram is not tri-colourable

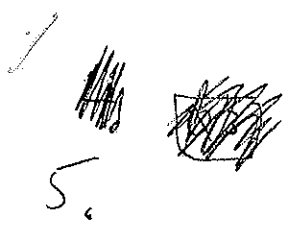


If P has only one colour (blue), then Q must be all blue, then R must be all blue, $\Rightarrow$ S all blue $\Rightarrow$ T all blue $\Rightarrow$ U all blue, so we do not have a tri-colouring.



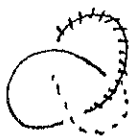
If P has all three colours: this is impossible as one arc appears twice at P.

Therefore the right hand diagram is not tri-colourable.



not tricolourable

(must use more than one colour)

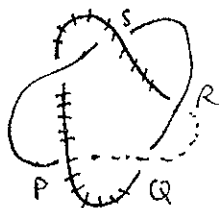


tricolourable (as in lecture)

4_1 = figure-8 - not tricolourable as in lectures



$S_1 =$



Case ① P has all 3 colours

$\Rightarrow$ so does Q $\Rightarrow$ so does R

this forces the +++ line at S and the --- line at S

$\Rightarrow$ two colours at S, not allowed.



Case ② P has only one colour $\Rightarrow$ same at Q $\Rightarrow$ same at R

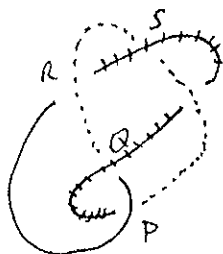


$\Rightarrow$ same at S $\Rightarrow$ all arcs are just one colour, not allowed.

$\therefore$ not tricolourable



$S_2 =$



Case ① P (say) has all 3 colours

$\Rightarrow$ same at Q $\Rightarrow$ same at R

$\Rightarrow$ only two colours at S - not allowed.

Case ② P has only one colour $\Rightarrow$ same at Q



$\Rightarrow$ same at R $\Rightarrow$ same at S

$\Rightarrow$ all arcs are just one colour, not allowed.

$\therefore$ not tricolourable

(so only 3_1

among ≤ 5 crossings is tricolourable)